PART A – CHEMISTRY (SET-C)

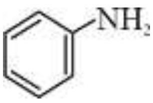
ALL THE GRAPHS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE

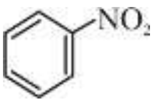
1. Which of the following salts is the most basic in aqueous solution?
(1) CH_3COOK (2) $FeCl_3$ (3) $Pb(CH_3COO)_2$ (4) $Al(CN)_3$

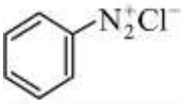
Solution:(1)

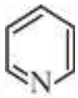
$CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-$
$Fe^{3+} + H_2O \rightleftharpoons Fe(OH)^{2+} + H^+$
$Pb^{2+} + H_2O \rightleftharpoons Pb(OH)^+ + H^+$
$CH_3COO^- + H_2O \rightleftharpoons CH_3COO^- + OH^-$
$Al^{3+} + H_2O \rightleftharpoons Al(OH)^{2+} + H^+$
$CN^- + H_2O \rightleftharpoons HCN + OH^-$

2. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

(1) 

(2) 

(3) 

(4) 

Solution:(1)

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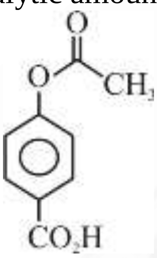
Kjeldahl's method is not applicable to compounds containing nitrogen in nitro and azo groups and nitrogen present in ring (Pyridine)

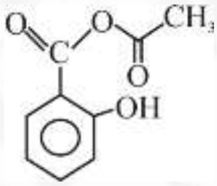
3. Which of the following are Lewis acids?
(1) $AlCl_3$ and $SiCl_4$ (2) PH_3 and $SiCl_4$ (3) BCl_3 and $AlCl_3$ (4) PH_3 and BCl_3

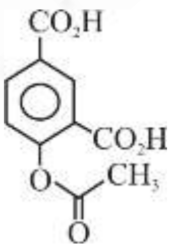
Solution:(3)

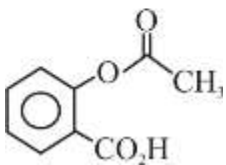
Lewis acids are electron deficient which can accept a lone pair of electron.

4. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces :

(1) 

(2) 

(3) 

(4) 

Solution:(4)

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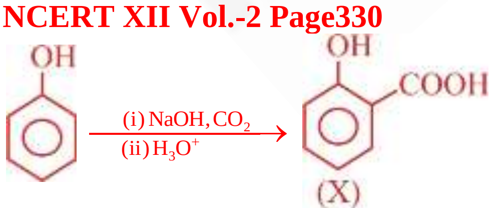

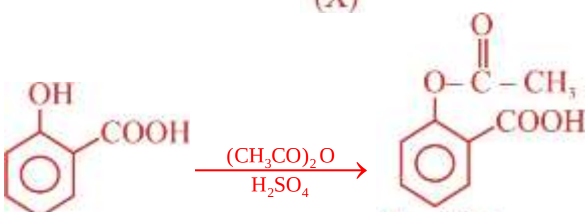

(Aspirin)

5. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

	Base	Acid	End point
(1)	Strong	Strong	Pinkish red to yellow
(2)	Weak	Strong	Yellow to pinkish red
(3)	Strong	Strong	Pink to colourless
(4)	Weak	Strong	Colourless to pink

Solution:(2)

Methyl orange shows red colour in acidic medium and yellow colour in basic medium.

6. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is :

(1) 3×10^{-20} (2) 6×10^{-21} (3) 5×10^{-19} (4) 5×10^{-8}

Solution:(1)

$$H_2S \rightleftharpoons 2H^+ + S^{2-}$$

$$[S^{2-}] = \frac{Ka_1 Ka_2}{[H^+]^2}[H_2S]$$

$$= \frac{1.2\times10^{-20}}{(0.2)^2}0.1 = 3\times10^{-20}$$

7. The combustion of benzene (*l*) gives $CO_2(g)$ and $H_2O(l)$. Given that heat of combustion of benzene at constant volume is – 3263.9 kJ mol^{-1} at 25°C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be :

(R = 8.314 JK^{-1} mol^{-1})

(1) –452.46 (2) 3260 (3) – 3267.6 (4) 4152.6

Solution:(3)

$$C_6H_{6(\ell)} + \frac{15}{2}O_{2(g)} \longrightarrow 6CO_{2(g)} + 3H_2O_{(\ell)}$$

$\Delta H = \Delta E + \Delta n_g RT$

$\Delta n_g = -3/2$

$$\Delta H = -3263.9 - \frac{3}{2}\times\frac{8.31\times298}{1000} = -3267.6$$

8. The compound that does not produce nitrogen gas by the thermal decomposition is :

(1) $(NH_4)_2Cr_2O_7$ (2) NH_4NO_2 (3) $(NH_4)_2SO_4$ (4) $Ba(N_3)_2$

Solution:(3)

$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + H_2O$

$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

$Ba(N_3)_2 \xrightarrow{\Delta} Ba + N_2$

9. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 of diborane?
(Atomic weight of B = 10.8 u)
(1) 0.8 hours (2) 3.2 hours (3) 1.6 hours (4) 6.4 hours

Solution:(2)

$B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

$$\frac{27.66}{27.6} = 1\text{mole}$$

no. of moles of O_2 required for oxidation = 3

For 1 mole O_2 from water needs 4F charge

$$\frac{100 \times t}{96500} = 12 \qquad \Rightarrow t = 3.2 \text{ hours}$$

10. Total number of lone pair of electrons in I_3^- ion is:
(1) 6 (2) 9 (3) 12 (4) 3

Solution:(2)

So 9 lone pairs

11. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :
(1) Ca (2) Al (3) Fe (4) Zn

Solution:(2)

$NaOH + Al \longrightarrow Al(OH)_3 + H_2$

$Al(OH)_3 + NaOH \longrightarrow NaAlO_2$

$2Al(OH)_3 \xrightarrow{\Delta} Al_2O_3 + 3H_2O$

Al_2O_3 used in chromatography as an adsorbent.

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12. According to molecular orbital theory, which of the following will not be a viable molecule?
(1) He_2^+ (2) H_2^- (3) H_2^{2-} (4) He_2^{2+}

Solution:(3)

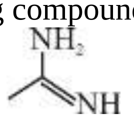
He_2^+ $\sigma 1s^2$ $\sigma^* 1s^1$ B.O.=0.5

H_2^- $\sigma 1s^2$ $\sigma^* 1s^1$ B.O.=0.5

H_2^{2-} $\sigma 1s^2$ $\sigma^* 1s^2$ B.O.=0

He_2^{2+} $\sigma 1s^2$ B.O.=1

13. The increasing order of basicity of the following compound is :

(a)  (b)  (c)  (d) 

(1) (b) < (a) < (c) < (d) (2) (b) < (a) < (d) < (c)

(3) (d) < (b) < (a) < (c) (4) (a) < (b) < (c) < (d)

Solution:(2)

14. Which type of 'defect' has the presence of cations in the interstitial sites?

(1) Vacancy defect (2) Frenkel defect

(3) Metal deficiency defect (4) Schottky defect

Solution:(2)

In Frenkel defect cation leaves its lattice site & moves to interstitial sites.

15. Which of the following compounds contain(s) no covalent bond(s)?

KCl, PH_3, O_2, B_2H_6, H_2SO_4

(1) KCl, H_2SO_4 (2) KCl (3) KCl, B_2H_6 (4) KCl, B_2H_6, PH_3

Solution:(2)

KCl have ionic bond only K^+ Cl^-

16. The oxidation states of

Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are :

(1) + 3, + 2, and +4 (2) + 3, 0, and +6 (3) +3, 0, and + 4 (4) +3, +4, and +6

Solution:(2)

$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$

$+ 2 + x - 2 - 4 - 2 = 0$

$x = + 6$

17. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively :

(1) $(H_2O + O_2)$ and $(H_2O + OH^-)$ (2) H_2O and $(H_2O + O_2)$

(3) H_2O and $(H_2O + OH^-)$ (4) $(H_2O + O_2)$ and H_2O

Solution:(2)

$$\overset{+2}{[Fe(CN)_6]^{4-}} + H_2\overset{1-}{O_2} + H^+ \longrightarrow \overset{+3}{[Fe(CN)_6]^{3-}} + H_2\overset{-2}{O}$$

$$\overset{+3}{[Fe(CN)_6]^{3-}} + H_2\overset{1-}{O_2} + OH^- \longrightarrow \overset{+2}{[Fe(CN)_6]^{4-}} + H_2O + \overset{0}{O_2}$$

18. Glucose on prolonged heating with HI gives:

(1) 1-Hexene (2) Hexanoic acid (3) 6-iodohexanal (4) n-Hexane

Solution:(4)

$$\underset{\text{(Glucose)}}{C_6H_{12}O_6} + HI \longrightarrow \text{n hexane}$$

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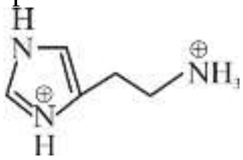
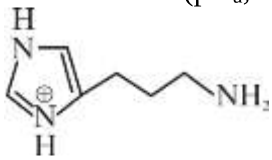
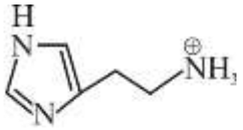
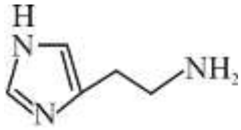
19. The predominant form of histamine present in human blood is (pK_a, Hisdidine = 6.0)

(1) 

(2) 

(3) 

(4) 

Solution:(3)

pH of human blood is 7.35 to 7.45

Histamine is alkaline with respect to human blood so structure is  .

20. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2.Ca(OH)_2]$ to :

(1) $[3(CaF_2).Ca(OH)_2]$
(2) $[3Ca_3(PO_4)_2.CaF_2]$
(3) $[3\{Ca(OH)_2\}.CaF_2]$
(4) $[CaF_2]$

Solution:(2)

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$3Ca_3(PO_4)_2.\ Ca(OH)_2 \longrightarrow 3Ca_3(PO_4)_2.CaF_2$

21. Consider the following reaction and statements :

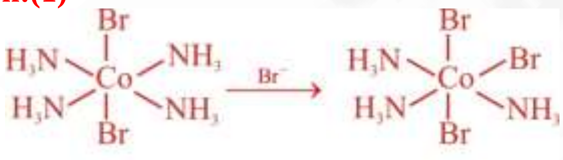
$$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$$

(I) Two isomers are produced if the reactant complex ion is a cis-isomer.
(II) Two isomers are produced if the reactant complex ion is a trans-isomer.
(III) Only one isomer is produced if the reactant complex ion is a trans-isomer.
(IV) Only one isomer is produced if the reactant complex ion is a cis-isomer.

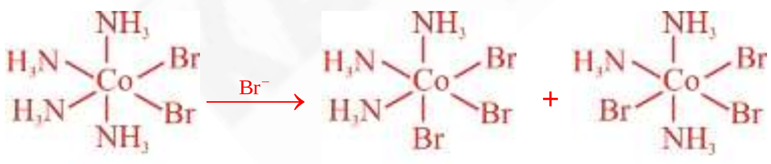
The correct statements are :

(1) (I) and (III) (2) (III) and (IV) (3) (II) and (IV) (4) (I) and (II)

Solution:(1)



(Trans)



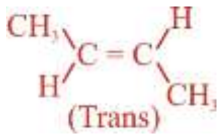
(Cis) (Fac) (mer-)

22. The trans-alkenes are formed by the reduction of alkynes with :

(1) $NaBH_4$ (2) Na/liq. NH_3

(3) Sn – HCl (4) H_2 – Pd/C, $BaSO_4$

Solution:(2)

$CH_3 - C \equiv C - CH_3 \xrightarrow[\text{liq. NH}_3]{\text{Na}}$ 

23. The ratio mass percent of C and H of an organic compound ($C_XH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_XH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of Compound $C_XH_YO_Z$ is :

(1) C_2H_4O (2) $C_3H_4O_2$ (3) $C_2H_4O_3$ (4) $C_3H_6O_3$

Solution:(3)

$$C_XH_Y + \left(X + \frac{Y}{4}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

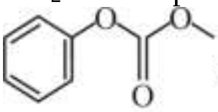
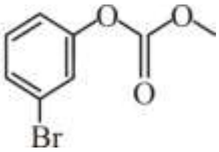
$$C_ZH_YO_Z + \left(X + \frac{Y}{4} - \frac{Z}{2}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

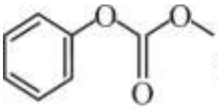
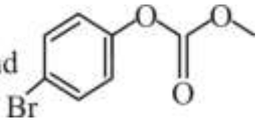
$$X + \frac{Y}{4} = Z$$

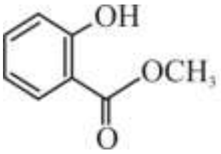
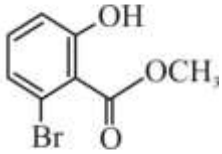
$$X : Y = \frac{6}{12} : 1 \Rightarrow 1 : 2$$

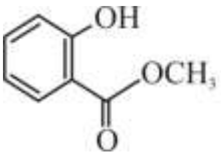
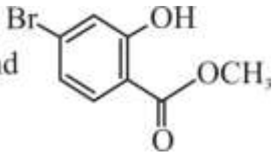
$$X : Y : Z = 2 : 4 : 3$$

24. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively :

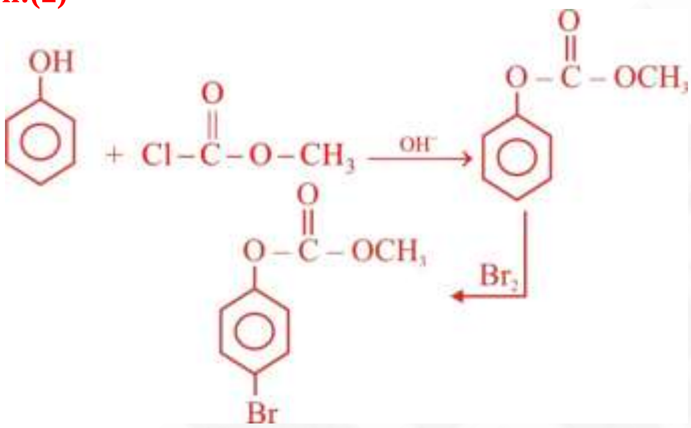
(1)  and 

(2)  and 

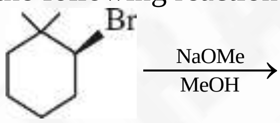
(3)  and 

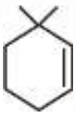
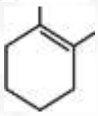
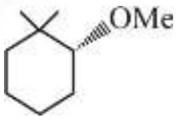
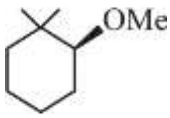
(4)  and 

Solution:(2)

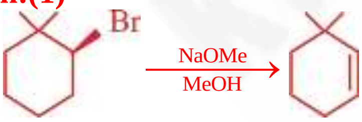


25. The major product of the following reaction is :

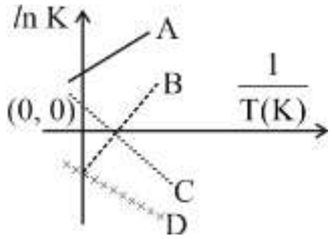


(1)  (2)  (3)  (4) 

Solution:(1)



26. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction?



(1) B and C (2) C and D (3) A and D (4) A and B

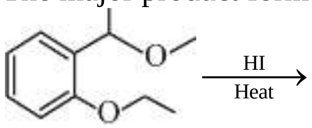
Solution:(4)

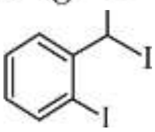
For exothermic reaction on increasing temperature equilibrium constant will decrease.

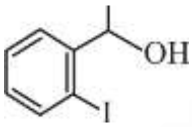
$$ln\ K = lnA - \frac{\Delta H}{RT}$$

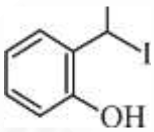
for exothermic reaction $\Delta H < 0$

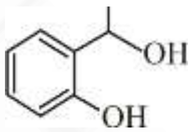
27. The major product formed in the following reaction is :



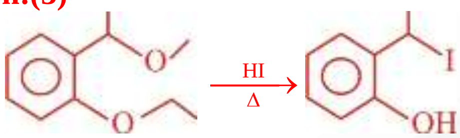
(1) 

(2) 

(3) 

(4) 

Solution:(3)



28. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+}?

(1) 2×10^{-9} M (2) 1.1×10^{-9} M (3) 1.0×10^{-10} M (4) 5×10^{-9} M

Solution:(2)

$[SO_4^{2-}] = 0.1$

$\left[Ba^{2+}\right] \times 0.1 = 1 \times 10^{-10}$

$\left[Ba^{2+}\right] = 1 \times 10^{-9} M$

$C \times 450 = 1 \times 10^{-9} \times 500$

$C = 1.1 \times 10^{-9}$ M

29. At 518° C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is :

(1) 3 (2) 1 (3) 0 (4) 2

Solution:(4)

Rate = $k\ p^n$

$1 = k\ [363 \times 0.95]^n$

$0.5 = k\ [363 \times 0.67]^n$

$$\left[\frac{0.95}{0.67}\right]^n = 2$$

$n = 2$

30. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_5Cl]Cl_2.H_2O$ (2) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

(3) $[Co(H_2O)_3Cl_3].3H_2O$ (4) $[Co(H_2O)_6]Cl_3$

Solution:(3)

The one having least number of units in aq. medium have highest freezing point.

PART B – MATHEMATICS (SET-C)

ALL THE GRAPHS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE

31. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x)^2}dx$ is equal to:

(1) $\frac{-1}{3(1+\tan^3 x)}+C$ (2) $\frac{1}{1+\cot^3 x}+C$ (3) $\frac{-1}{1+\cot^3 x}+C$ (4) $\frac{1}{3(1+\tan^3 x)}+C$

(where C is a constant of integration)

Solution: (1)

$$\int \frac{\frac{\sin^2 x\cos^2 x}{\cos^{10} x}}{(\tan^5 x+\tan^2 x+\tan^3 x+1)^2}dx$$

$$=\int \frac{\tan^2 x.\sec^6 x}{(\tan^3 x+1)^2(\sec^2 x)^2}dx$$

$$=\int \frac{\tan^2 x.\sec^2 x}{(\tan^3 x+1)^2}dx$$

Put $\tan^3 x+1=t$

$\Rightarrow 3\tan^2 x\sec^2 x\,dx=dt$

Now, $\frac{1}{3}\int\frac{dt}{t^2}=\frac{-1}{3t}+C \Rightarrow -\frac{1}{3(1+\tan^3 x)}+C$

32. Tangents are drawn to the hyperbola $4x^2-y^2=36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is:

(1) $54\sqrt{3}$ (2) $60\sqrt{3}$ (3) $36\sqrt{5}$ (4) $45\sqrt{5}$

Solution: (4)

Equation of chord of contact from (0, 3) to the given hyperbola is

$4.(0)x-y3-36=0$

$3y=-36$

$y=-12$

Solving y = –12 and $4x^2 - y^2 = 36$ for point P and Q, we get

$4x^2-144=36$

$4x^2=180$

$x=\pm 3\sqrt{5}$

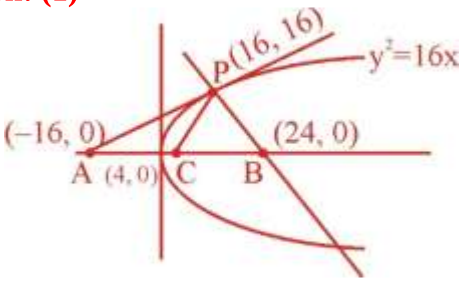
$P(3\sqrt{5},-12)\ Q(-3\sqrt{5},-12)\ \ T(0,3)$

Area of $\Delta PQT=\frac{3\sqrt{5}\times 15\times 2}{2}=45\sqrt{5}$

33. Tangent and normal are drawn at $P(16,16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is:

(1) 2 (2) 3 (3) $\frac{4}{3}$ (4) $\frac{1}{2}$

Solution: (1)



Equation of $PA = 2y = x + 16$ and $PB = 2x + y = 48$

$\therefore\ A = (-16, 0),\ B = (24, 0)$ and $C(4,\ 0)$

$$m_{PB} = \frac{16}{-8} = -2$$

$$m_{PC} = \frac{4}{3}$$

$$\tan\theta = \left|\frac{\frac{4}{3}+2}{1-\frac{8}{3}}\right| = \frac{10}{5} = 2$$

34. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b} = 24$, then $|\vec{u}|^2$ is equal to:

(1) 315 (2) 256 (3) 84 (4) 336

Solution: (4)

$$\bar{u} = \lambda(2\hat{i} + 3\hat{j} - \hat{k}) + \mu(\hat{j} + \hat{k})$$

$$\bar{u} = (2\lambda)\hat{i} + (3\lambda + \mu)\hat{j} + (-\lambda + \mu)\hat{k}$$

$$\bar{u}.\bar{a} = 2(2\lambda) + 3(3\lambda + \mu) - (-\lambda + \mu) = 0$$

$$\bar{u}.\bar{b} = (3\lambda + \mu) + (-\lambda + \mu) = 24$$

$$14\lambda + 2\mu = 0 \quad \ldots (1)$$

$$2\lambda + 2\mu = 24 \quad \ldots (2)$$

Solving (1) and (2), we get

$\lambda = -2,\ \mu = +14$

$$\bar{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$$

$$|\bar{u}| = \sqrt{16 + 64 + 256} = \sqrt{336}$$

35. If α, $\beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to:

(1) 0 (2) 1 (3) 2 (4) –1

Solution: (2)

$x^2 - x + 1 = 0$

$x = -\omega, -\omega^2$, (where ω is an imaginary cube root of unity)

$\alpha^{101} + \beta^{107} \Rightarrow (-\omega)^{101} + (-\omega^2)^{107} \Rightarrow -\omega^2 - \omega = 1$

36. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and α, $\beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and y = 0, is:

(1) $\frac{1}{2}\left(\sqrt{3}+1\right)$ (2) $\frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$ (3) $\frac{1}{2}\left(\sqrt{2}-1\right)$ (4) $\frac{1}{2}\left(\sqrt{3}-1\right)$

Solution: (4)

$18x^2 - 9\pi x + \pi^2 = 0$

$18x^2 - 6\pi x - 3\pi x + \pi^2 = 0$

$6x(3x - \pi) - \pi - (3x - \pi) = 0$

$x = \frac{\pi}{3}$ and $\frac{\pi}{6}$

$\beta = \frac{\pi}{3}$ and $\alpha = \frac{\pi}{6}$ $\quad (\beta > \alpha)$

$g(x) = \cos x^2$

$f(x) = \sqrt{x}$

$y = g(f(x)) \Rightarrow g(\sqrt{x}) = \cos x$

The area bounded by the curve $y = \cos x$ and lines $x = \frac{\pi}{6}$

$x = \frac{\pi}{3}$ and $y = 0$ is

$\int_{\pi/6}^{\pi/3} \cos x\, dx \Rightarrow \sin x\Big|_{\pi/6}^{\pi/3} = \frac{\sqrt{3}-1}{2}$

37. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is:-

(1) 0 (2) 1 (3) 2 (4) –1

Solution: (3)

$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$\Rightarrow 2\left({}^5C_o x^5 + {}^5C_2 x^3(x^3-1) + {}^5C_4 x(x^3-1)^2\right)$

Sum of all odd degree terms:

$2\left({}^5C_o - {}^5C_2 + {}^5C_4 + {}^5C_4\right) \Rightarrow 2(1-10+5+5) \Rightarrow 2$

38. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.
If $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to:
(1) 68 (2) 34 (3) 33 (4) 66

Solution: (2)

Let $a_1 = a$ and common difference = d

$\sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow a + 24d = 32$

$a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$

$\therefore a = 8,\ d = 1$

$\therefore\ a_n = n + 7$

$\sum_{i=1}^{17} a_n^2 = \sum_{i=1}^{17} (n+7)^2 = \sum n^2 + 14\sum n + 49\sum 1 = 17 \times 140 \times 2$

$\therefore\ 140m = 17 \times 140 \times 2$

$m = 34$

39. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots x_9$ is:
(1) 4 (2) 2 (3) 3 (4) 9

Solution: (2)

$\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$

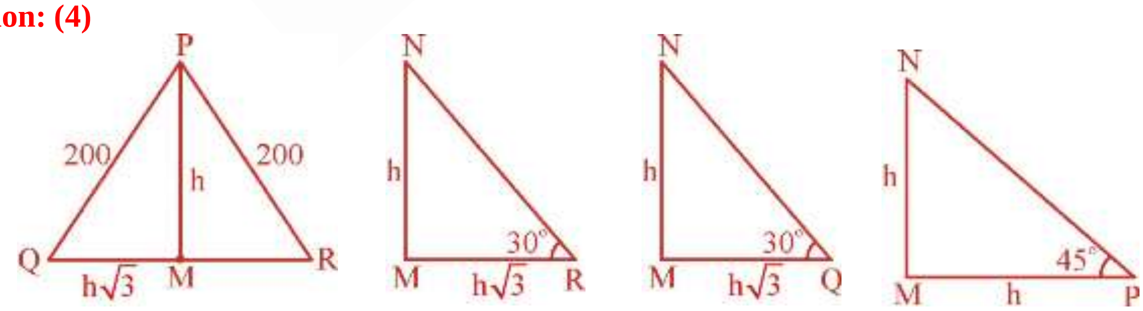
$\sum x_i = 54$ and $\sum (x_i^2 + 25 - 10x_i) = 45$

$\sum x_i = 54$ and $\sum x_i^2 = 360$

$\therefore$ S.D. $= \sqrt{\dfrac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\dfrac{\sum x_i^2}{n} - \bar{x}^2} = \sqrt{40 - 36} = 2$

40. PQR is a triangular park with $PQ = PR = 200\,m$. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is:
(1) 50 (2) $100\sqrt{3}$ (3) $50\sqrt{2}$ (4) 100

Solution: (4)



$h^2 + 3h^2 = 40000$

$h = 100$

41. Two sets A and B are as under:

$A=\{(a,b)\in \mathbf{R}\times\mathbf{R}:|a-5|<1 \text{ and } |b-5|<1\}$; $B=\{(a,b)\in\mathbf{R}\times\mathbf{R}:4(a-6)^2+9(b-5)^2\le 36\}$.

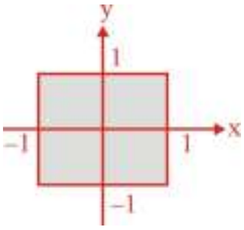
Then:

(1) $A\subset B$
(2) $A\bigcap B=\phi$ (an empty set)
(3) neither $A\subset B$ nor $B\subset A$
(4) $B\subset A$

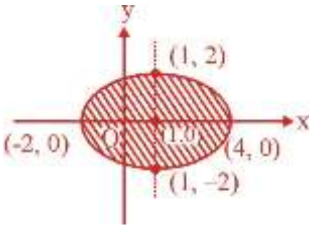
Solution: (1)

Let $a-5=x$, $b-5=y$

$A:|x|<1$ and $|y|<1$



$B:4(x-1)^2+9y^2\le 36$



$\therefore\ A\subset B$

42. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is:

(1) less than 500
(2) at least 500 but less than 750
(3) at least 750 but less than 1000
(4) at least 1000

Solution: (4)

No. of ways to choose = ${}^6C_4\,{}^3C_1=45$

No. of ways to arrange = $\underline{|4}=24$

Total ways = $45\times 24=1080$

43. Let $f(x)=x^2+\frac{1}{x^2}$ and $g(x)=x-\frac{1}{x}, x\in\mathbf{R}-\{-1,0,1\}$. If $h(x)=\frac{f(x)}{g(x)}$, then the local minimum value of h(x) is:

(1) –3 (2) $-2\sqrt{2}$ (3) $2\sqrt{2}$ (4) 3

Solution: (3)

$$h(x)=\frac{f(x)}{g(x)}=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}=\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}$$

If $\left(x-\frac{1}{x}\right)>0$, then $h(x)\geq 2\sqrt{2}$ [using $AM\geq GM$]

Similarly if $\left(x-\frac{1}{x}\right)<0$, then $h(x)\leq -2\sqrt{2}$

44. For each $t\in\mathbf{R}$, let [t] be the greatest integer less than or equal to t. Then $\lim\limits_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)$

(1) is equal to 15 (2) is equal to 120
(3) does not exist (in R) (4) is equal to 0

Solution: (2)

$$\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)=\lim_{x\to 0^+} x\left(\frac{15\times 16}{2x}-\left\{\frac{1}{x}\right\}-\left\{\frac{2}{x}\right\}\ldots\left\{\frac{15}{x}\right\}\right)$$

$$=\lim_{x\to 0^+} 120-x\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+\ldots\left\{\frac{15}{x}\right\}\right)=120$$

45. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin^2 x}{1+2^x}dx$ is:

(1) $\frac{\pi}{2}$ (2) 4π (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

Solution: (3)

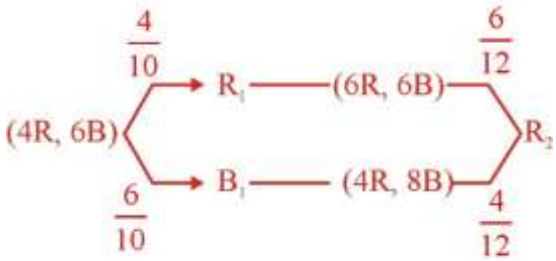
$$I=\int_{-\pi/2}^{\pi/2}\frac{\sin^2 x}{1+2^x}dx=\int_0^{\pi/2}\frac{\sin^2 x}{1+2^x}+\frac{\sin^2 x}{1+2^{-x}}dx$$

$$=\int_0^{\pi/2}\sin^2 x\,dx=\int_0^{\pi/2}\frac{1-\cos 2x}{2}dx=\frac{x}{2}-\frac{\sin 2x}{4}\Big|_0^{\pi/2}=\frac{\pi}{4}$$

46. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{3}{4}$ (4) $\frac{3}{10}$

Solution: (1)



P (second ball is red) $= P(R_2) = \left(\frac{4}{10}\times\frac{6}{12}\right)+\left(\frac{6}{10}\times\frac{4}{12}\right)=\frac{2}{5}$

47. The length of the projection of the line segment joining the points $(5,-1,4)$ and $(4,-1,3)$ on the plane, $x+y+z=7$ is:

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{\sqrt{3}}$

Solution: (3)

Let A(5, –1, 4) and B (4, –1, 3)

$\overrightarrow{BA}=\hat{i}+\hat{k}$

Projection of $\overrightarrow{BA}$ along normal to plane is $=\frac{(\hat{i}+\hat{k})\,.\,(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}=\frac{2}{\sqrt{3}}$

Required projection $=\sqrt{2-\frac{4}{3}}=\sqrt{\frac{2}{3}}$

48. If sum of all the solutions of the equation $8\cos x\left(\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0,\pi]$ is $k\pi$, then k is equal to:

(1) $\frac{13}{9}$ (2) $\frac{8}{9}$ (3) $\frac{20}{9}$ (4) $\frac{2}{3}$

Solution: (1)

$$8\cos x\left(\cos^2 x-\sin^2\frac{\pi}{6}-\frac{1}{2}\right)=1$$

$$8\cos x\left(\cos^2 x-\frac{3}{4}\right)=1$$

$$2\cos x\left(4\cos^2 x-3\right)=1$$

$$\cos 3x=\frac{1}{2}$$

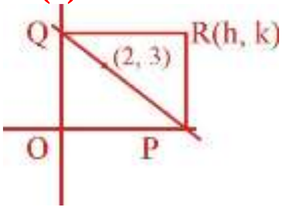
Required solutions are $\frac{\pi}{9},\left(\frac{2\pi}{3}-\frac{\pi}{9}\right),\left(\frac{2\pi}{3}+\frac{\pi}{9}\right)$

Sum of all the solutions $=\frac{13\pi}{9}$

49. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

(1) $2x+3y=xy$ (2) $3x+2y=xy$ (3) $3x+2y=6xy$ (4) $3x+2y=6$

Solution: (2)



Let $R(h,k)$

Then $P(h,0)$ and $Q(0,k)$

$\therefore$ Equation of $PQ=\frac{x}{h}+\frac{y}{k}=1$

PQ passes through (2, 3)

$\frac{2}{h}+\frac{3}{k}=1$

$\therefore$ Locus $2y+3x=xy$

50. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+...$ If $B-2A=100\lambda$, then λ is equal to:

(1) 248 (2) 464 (3) 496 (4) 232

Solution: (1)

$S=1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+...2n\text{ terms}$

$=(1^2+3^2+5^2+...n\text{ terms})+2.2^2\left[(1^2+2^2+3^2+...n\text{ terms})\right]$

$=\sum(2n-1)^2+8\sum n^2=4n^2(n+1)+n$

$\therefore$ $A=S_{20}=400\times11+10=4410$

$B=S_{40}=4(400)(21)+20=33620$

$B-2A=24800$

$\lambda=248$

51. If the curves $y^2=6x$, $9x^2+by^2=16$ intersect each other at right angles, then the value of b is:

(1) $\frac{7}{2}$ (2) 4 (3) $\frac{9}{2}$ (4) 6

Solution: (3)

Let (x, y) be the point of intersection, then

$y^2=6x$ … (1)

$9x^2+by^2=16$ … (2)

(1) and (2) are perpendicular

$\frac{6}{2y}\times\frac{-18x}{2by}=-1$

$\frac{27x}{by^2}=1=\frac{27x}{b(6x)}$

$b=\frac{27}{6}=\frac{9}{2}$

52. Let the orthocentre and centroid of a triangle be $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$ (3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

Solution: (2)

Orthocenter = $A(-3,5)$

Centroid = $B(3,3)$

Centroid divides the line joining orthocentre and circumcenter in ratio 2 : 1

Circumcenter = C(6, 2) [Using section formula]

Radius of circle having AC as diameter $= \frac{AC}{2} = 3\sqrt{\frac{5}{2}}$

53. Let $S = \left\{t \in R : f(x) = |x-\pi| . \left(e^{|x|}-1\right)\sin|x| \text{ is not differentiable at } t\right\}$. Then the set S is equal to:

(1) $\{0\}$ (2) $\{\pi\}$ (3) $\{0, \pi\}$ (4) ϕ (an empty set)

Solution: (4)

$f(x) = |x-\pi|\left(e^{|x|}-1\right)\sin|x|$

Checking at $x = 0$

$$f(x) = \begin{cases} (\pi - x)(e^{x}-1)\sin x, & x \to 0^{+} \\ (\pi - x)(e^{-x}-1)(-\sin x), & x \to 0^{-} \end{cases}$$

$\therefore f'(0) = 0$

Checking at $x = \pi$

$$f(x) = \begin{cases} (x-\pi)(e^{x}-1)\sin x, & x \to \pi^{+} \\ -(x-\pi)(e^{x}-1)\sin x, & x \to \pi^{-} \end{cases}$$

$\therefore$ At $x = \pi$, $RHD = (e^{\pi}-1)\sin\pi = 0$; $LHD = -(e^{\pi}-1)\sin\pi = 0$

$\therefore$ Function f(x) is always differentiable $\forall\ x \in R$.

54. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to;

(1) $(-4,3)$ (2) $(-4,5)$ (3) $(4,5)$ (4) $(-4,-5)$

Solution: (2)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix} \quad C_1 \to C_1 + C_2 + C_3$$

$$= (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x-4 & 0 \\ 0 & 0 & -x-4 \end{vmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$= (5x-4)(x+4)^2$

A = –4, B = 5

55. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to:

(1) p (2) q (3) ~q (4) ~p

Solution: (4)

$\sim(p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge (\sim q \vee q)$

$\equiv \sim p \wedge t$

$\equiv \sim p$

56. If the system of linear equations

$$x + ky + 3z = 0$$
$$3x + ky - 2z = 0$$
$$2x + 4y - 3z = 0$$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to:

(1) 10 (2) –30 (3) 30 (4) –10

Solution: (1)

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow -k(-5) + k(-9) - 4(-11) = 0$

$5k - 9k + 44 = 0$

$k = 11$

$x + 11y + 3z = 0$ ... (1)

$3x + 11y - 2z = 0$ ... (2)

$2x + 4y - 3z = 0$ ... (3)

$\frac{x}{y} + 3\frac{z}{y} + 11 = 0$

$3\frac{x}{y} - 2\frac{z}{y} + 11 = 0$

$\therefore \frac{x}{y} = -5, \frac{z}{y} = -2$

$\therefore \frac{xz}{y^2} = 10$

57. Let $S = \left\{x \in \mathbf{R} : x \geq 0 \text{ and } 2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0\right\}$. Then S:

(1) contains exactly one element
(2) contains exactly two elements
(3) contains exactly four elements
(4) is an empty set

Solution: (2)

$$2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0$$

$$2\left|\sqrt{x} - 3\right| + \left(\sqrt{x}\right)^2 - 6\sqrt{x} + 6 = 0$$

$$2\left|\sqrt{x} - 3\right| + \left(\sqrt{x} - 3\right)^2 = 3$$

Put $\left|\sqrt{x} - 3\right| = t$

$\Rightarrow t^2 + 2t - 3 = 0$

$\Rightarrow t = 1, -3$

$\Rightarrow \left|\sqrt{x} - 3\right| = 1$

$\Rightarrow \sqrt{x} = 3 \pm 1$

$\Rightarrow x = 16, 4$

58. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is:

(1) 185
(2) 85
(3) 95
(4) 195

Solution: (3)

Equation of tangent is $x(1) = \frac{y+7}{2} - 6$

$\Rightarrow 2x = y + 7 - 12 = y - 5$

$\Rightarrow 2x - y + 5 = 0$

It touches $(x+8)^2 + (y+6)^2 = 100 - c$

$\therefore \left|\frac{-16 + 6 + 5}{\sqrt{5}}\right| = \sqrt{100 - c}$

$\Rightarrow 100 - c = 5$

$\Rightarrow c = 95$

59. Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y\cos x = 4x,\ x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(1) $\frac{-8}{9\sqrt{3}}\pi^2$ (2) $-\frac{8}{9}\pi^2$ (3) $-\frac{4}{9}\pi^2$ (4) $\frac{4}{9\sqrt{3}}\pi^2$

Solution: (2)

$\sin x \frac{dy}{dx} + y\cos x = 4x$

$\sin x dy + y\cos x dx = 4x\ dx$

$(\sin x)dy + y\ d(\sin x) = 4x\ dx$

$\int d(y\sin x) = \int 4x\,dx$

$y\sin x = 2x^2 + c$

Putting $\left(\frac{\pi}{2}, 0\right)$ we get $c = -\frac{\pi^2}{2}$

$\therefore\ y\sin x = 2x^2 - \frac{\pi^2}{2}$

Now $y\left(\frac{\pi}{6}\right) = \left(2\left(\frac{\pi^2}{36}\right) - \frac{\pi^2}{2}\right)2 = \frac{\pi^2}{9} - \pi^2 = -\frac{8\pi^2}{9}$

60. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2, is:

(1) $\frac{1}{3\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4\sqrt{2}}$

Solution: (1)

Family of planes containing the line L_1 is

$$(2x - 2y + 3z - 2) + \lambda(x - y + z + 1) = 0 \qquad \ldots (1)$$

Family of plane containing the line L_2 is

$$(x + 2y - z - 3) + \mu(3x - y + 2z - 1) = 0 \qquad \ldots (2)$$

Equating planes (1) and (2), we get a plane containing both lines L_1 and L_2

$$\frac{2+\lambda}{1+3\mu} = \frac{-2-\lambda}{2-\mu} = \frac{3+\lambda}{-1+2\mu} = \frac{-2+\lambda}{-3-\mu}$$

Solving, we get $\lambda = 5$, $\mu = -\frac{3}{2}$

$\therefore$ Equation of required plane is $7x - 7y + 8z + 3 = 0$

Distance from origin is $\frac{3}{\sqrt{49+49+64}} = \frac{3}{9\sqrt{2}} = \frac{1}{3\sqrt{2}}$

PART C – PHYSICS (SET-C)

ALL THE GRAPHS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE

61. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near to it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?
(1) 50μm (2) 75μm (3) 100μm (4) 25μm

Solution: (4)

Angular width in single slit diffraction pattern = $(\Delta\theta) = 2\sin^{-1}\left(\frac{\lambda}{b}\right)$

$= 5 \times 10^{-7}$ m.

In YDSE,

Fringe width, $W = \frac{\lambda D}{d}$

$10^{-2} = \frac{(5\times10^{-7})(0.5)}{d}$

$d = 2.5 \times 10^{-5} = 25\mu m$.

62. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state then for large n, (A, B are constants)

(1) $\Lambda_n \approx A + B\lambda_n$ (2) $\Lambda_n^2 \approx A + B\lambda_n^2$ (3) $\Lambda_n^2 \approx \lambda$ (4) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

Solution: (4)

De-Broglie Wavelength is given by

$\lambda_n = \frac{h}{\sqrt{2m\,E_n}} = \frac{nh}{\sqrt{2m\,E_1}}$

$\lambda_g = \frac{h}{\sqrt{2m\,E_1}}$

hence $\Delta E = E_1 - E_n = E_1 - \frac{E_1}{n^2}$

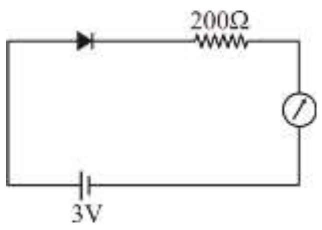
$\Lambda_n = \frac{hc}{E_1\left(1-\frac{1}{n^2}\right)}$

$\cong \frac{hc}{E_1}\left(1+\frac{1}{n^2}\right)$

$= \frac{hc}{E_1} + \frac{hc}{n^2E_1}$

$= A + \frac{B}{\lambda_n^2}$.

63. The reading of the ammeter for a silicon diode in the given circuit is:
(1) 15 mA
(2) 11.5 mA
(3) 13.5 mA
(4) 0



Solution: (2)

Using KVL,

$$-3 + 0.7 + I(200) = 0$$

$$I = \frac{2.3}{200} = 11.5\,\text{mA}.$$

64. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is:
(1) 3.5% (2) 4.5% (3) 6% (4) 2.5%

Solution: (2)

Density (d) = $\frac{m}{a^3}$

$$\frac{\Delta d}{d} = \frac{\Delta m}{m} + \frac{3\Delta a}{a}$$

$$= 1.5 + 3 \times 1 = 4.5\%.$$

65. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is:
(1) $r_e < r_p = r_\alpha$ (2) $r_e < r_p < r_\alpha$ (3) $r_e < r_\alpha < r_p$ (4) $r_e > r_p = r_\alpha$

Solution: (1)

for a particle moving in a circular path in external magnetic field,

$$r = \frac{mv}{qB} = \frac{\sqrt{2mKE}}{qB}$$

and $$r_e = \frac{\sqrt{2m_eKE}}{eB} \propto \frac{\sqrt{m_e}}{e}$$

$$r_p = \frac{\sqrt{2m_pKE}}{eB} \propto \frac{\sqrt{m_p}}{e}$$

$$r_\alpha = \frac{\sqrt{4m_pKE}}{2eB} \propto \frac{\sqrt{m_p}}{e}.$$

66. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is:

(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$ (2) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (3) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$ (4) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

Solution: (1)

$q_A = \sigma(4\pi a^2)$

$q_B = -\sigma(4\pi b^2)$

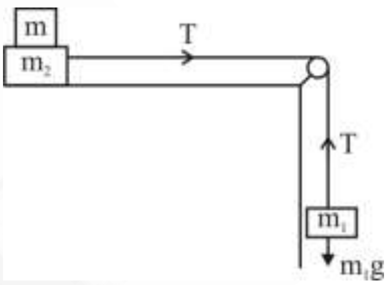
$q_C = \sigma(4\pi c^2)$

$$V_B = \frac{k(q_A+q_B)}{b}+\frac{kq_c}{c}$$

$$= \frac{1}{4\pi \epsilon_0}\sigma(4\pi)\left[\frac{a^2-b^2}{b}+c\right]$$

$$= \frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right].$$

67. Two masses m_1 = 5kg and m_2 = 10 kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(1) 27.3 kg
(2) 43.3 kg
(3) 10.3 kg
(4) 18.3 kg

Solution: (1)

The block system will be at rest if maximum friction is greater than the pulling force.

$\mu\,(m + m_2)g = m_1g$

$0.15\,(m + 10) = 5$

M = 23.3 kg.

68. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is:

(1) $\frac{k}{2a^2}$ (2) Zero (3) $-\frac{3}{2}\frac{k}{a^2}$ (4) $-\frac{k}{4a^2}$

Solution: (2)

$$F = -\frac{dU}{dr} = \frac{k}{2}\left[\frac{-2}{r^3}\right]$$

Hence,

$$\frac{k}{r^3} = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv^2 = \frac{k}{r^2}$$

$$KE = \frac{1}{2}mv^2 = \frac{k}{2r^2}$$

$$U = -\frac{k}{2r^2}$$

E = KE + U = 0.

69. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be:

(1) 0.3 nC (2) 2.4 nC (3) 0.9 nC (4) 1.2 nC

Solution: (4)

$$C_{new} = KC = \frac{5}{3}(90\,pF)$$

$$Q_{total} = C_{new}\,V = \frac{5}{3} \times 90 \times 20 = 3 \times 10^3\ pC$$

$$= 3\ nC$$

$$Q' = Q_{total}\left(1 - \frac{1}{k}\right)$$

$$= 3\left(1 - \frac{3}{5}\right) = 3 \cdot \frac{2}{5} = \frac{6}{5} = 1.2nC.$$

70. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 gm and Avagadro number = 6.02×10^{23} gm mole^{-1})

(1) 7.1 N/m (2) 2.2 N/m (3) 5.5 N/m (4) 6.4 N/m

Solution: (1)

$$T = 2\pi\sqrt{\frac{M}{k}} = \frac{1}{f}$$

$$K = (4\pi^2)f^2 m$$

$$K = 7.07\ N/m.$$

71. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The value of p_d and p_c are respectively:

(1) (.28, .89) (2) (0, 0) (3) (0, 1) (4) (.89, .28)

Solution: (4)

For collision of neutron with deuterium

$$mv = 2mv_1 + mv_2$$

$$v = v_1 - v_2$$

Solving we get

$$v_1 = \frac{2v}{3}$$

$$p_d = \frac{\frac{1}{2}2mv_1^2}{\frac{1}{2}mv^2} = 0.89$$

For collision of neutron with carbon

$$v = 12v_1 + v_2$$

$$v = v_1 - v_2$$

Solving we get $v_1 = \frac{2v}{13}$.

$$\text{Hence } p_c = \frac{\frac{1}{2} \times 12m \times \frac{4v^2}{169}}{\frac{1}{2}mv^2} = 0.28.$$

72. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is:

(1) $\sqrt{3}$ (2) $\sqrt{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) 2

Solution: (2)

Dipole moment = Current × Area of loop
When moment is doubled hence area also doubled

$A = \pi r^2$

$2A = \pi r'^2$

$\Rightarrow \quad r' = \sqrt{2}r$

$B_1 = \frac{\mu_0 I}{2r}$

$B_2 = \frac{\mu_0 I}{2r'} = \frac{\mu_0 I}{2\sqrt{2}r}$

$\frac{B_1}{B_2} = \sqrt{2}$.

73. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5Ω, a balance is found when the cell is connected across 40cm of the wire. Find the internal resistance of the cell.

(1) 1.5Ω (2) 2Ω (3) 2.5Ω (4) 1Ω

Solution: (1)

$r = R\left(\frac{\ell_1}{\ell_2} - 1\right)$

$= 5\left(\frac{52}{40} - 1\right) = 1.5\Omega$.

74. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^4 (2) 2×10^5 (3) 2×10^6 (4) 2×10^3

Solution: (2)

$10\,\text{GHz} \xrightarrow{10\%} 1\,\text{GHz} = 1 \times 10^9\,\text{Hz}$

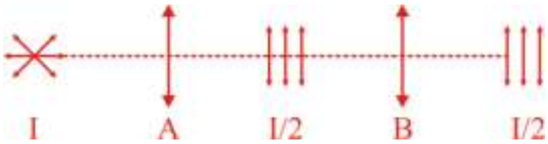
No. of channels

$= \frac{1 \times 10^9}{5 \times 10^3} = \frac{10^6}{5} = 2 \times 10^5$.

75. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found that be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B, The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is:

(1) 30° (2) 45° (3) 60° (4) 0°

Solution: (2)



Hence A and B are parallel.
Let C is kept at angle θ with A.
Then intensity after crossing

$$C = \frac{I}{2}\cos^2\theta$$

Again after crossing

$$B = \left[\frac{I}{2}\cos^2\theta\right]\cos^2(90-\theta) = \frac{I}{8}\sin^2 2\theta$$

$\therefore \quad \theta = 45°$.

76. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10cm. The resistance of their series combination is 1kΩ. How much was the resistance on the left slot before interchanging the resistances?

(1) 505Ω (2) 550Ω (3) 910Ω (4) 990Ω

Solution: (2)

$$\frac{R}{1000-R} = \frac{x}{100-x} \quad ...(i)$$

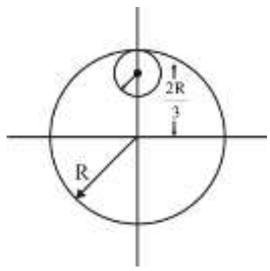
$$\frac{1000-R}{R} = \frac{x-10}{110-x} \quad ...(ii)$$

$$x - 10 = 100 - x$$

$$2x = 110$$

x = 55 cm and R = 550 Ω.

77. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is:



(1) $\frac{40}{9}MR^2$

(2) 10 MR^2

(3) $\frac{37}{9}MR^2$

(4) 4 MR^2

Solution: (4)

$$I_{res} = I_{total} - I_{removed}$$

$$I_{total} = \frac{(9M)R^2}{2}$$

$$I_{removed} = \frac{(M)(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2 = \frac{MR^2}{2}$$

hence $I_{res} = 4MR^2$.

78. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

(1) $\sqrt{2}v_0$ (2) $\frac{v_0}{2}$ (3) $\frac{v_0}{\sqrt{2}}$ (4) $\frac{v_0}{4}$

Solution: (1)

The increase of KE of may be due to some internal energy conversion. The increase will be shared between particles in C-frame.

$$\frac{1}{2}\mu V_{rel}^2 - \frac{1}{2}\mu V_0^2 = \frac{1}{2}\left(\frac{1}{2}mV_0^2\right)$$

$$V_{res} = \sqrt{2}V_0.$$

79. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z-ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivities of air and medium respectively, which of the following option is correct?

(1) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$ (2) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$ (3) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$ (4) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$

Solution: (2)

R.I. of medium – 1 $\Rightarrow \mu_1 = 1 = \frac{c}{V} = \frac{c}{c} = 1$

R.I. of medium – 2 $\Rightarrow \mu_2 = \frac{c}{V} = \frac{c}{c/2} = 2$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \text{ and } V = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}}$$

$$\Rightarrow \sqrt{\epsilon_r} = \frac{\mu_2}{\mu_1} = 2$$

$$\epsilon_{r_2} = 4\epsilon_{r_1}$$

$$\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}.$$

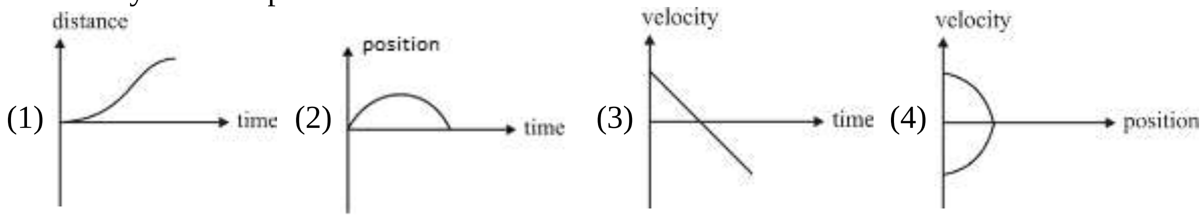
80. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exibits resonance. The quality factor, Q is given by:

(1) $\frac{\omega_0 R}{L}$ (2) $\frac{R}{(\omega_0 C)}$ (3) $\frac{CR}{\omega_0}$ (4) $\frac{\omega_0 L}{R}$

Solution: (4)

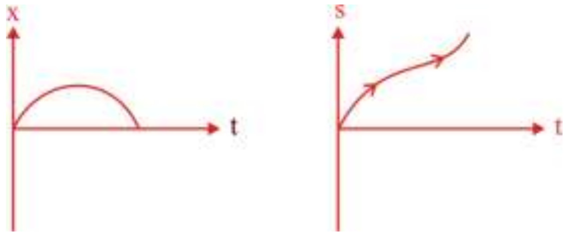
Quality factor = $\frac{\omega_0 L}{R}$.

81. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.



Solution: (1)

Correct distance time graph should be



82. Two batteries with e.m.f. 12V and 13V are connected in parallel across a load resistor is 10Ω. The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between:

(1) 11.5V and 11.6V (2) 11.4V and 11.5V
(3) 11.7V and 11.8V (4) 11.6V and 11.7V

Solution: (1)

$$\Delta V = E_{eq} = \left(\frac{\frac{12}{1}+\frac{13}{2}+\frac{0}{10}}{\frac{1}{1}+\frac{1}{2}+\frac{1}{10}}\right) = \left(\frac{18\frac{1}{2}}{1\frac{3}{5}}\right) = 11.5625\,\text{Volt}.$$

83. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then:

(1) $T \propto R^{\frac{n}{2}+1}$ (2) $T \propto R^{(n+1)/2}$ (3) $T \propto R^{n/2}$ (4) $T \propto R^{3/2}$ for any n

Solution: (2)

$$F \propto \frac{1}{R^n} = \frac{K}{R^n} = \frac{mv^2}{R}$$

$$V \propto \frac{1}{R^{\frac{n-1}{2}}}$$

$$T = \frac{2\pi R}{V} \propto R \cdot R^{\frac{(n-1)}{2}} \propto R^{\frac{n+1}{2}}.$$

84. If the series limit frequency of the Lyman series is v_L, then the series limit frequency of the Pfund series is:

(1) 16 v_L (2) $v_L/16$ (3) $v_L/25$ (4) 25 v_L

Solution: (3)

For Lyman

$$v_L \propto \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = 1$$

For Pfund

$$v_P \propto \left(\frac{1}{5^2} - \frac{1}{\infty}\right) = \frac{1}{25}$$

$$\frac{v_P}{v_L} = \frac{1}{25}$$

$$\Rightarrow \quad v_p = \frac{v_L}{25}.$$

85. If an a.c. circuit, the instantaneous e.m.f. and current are given by

e = 100 sin 30t

$$i = 20\sin\left(30t - \frac{\pi}{4}\right)$$

In one cycle of a.c., the average power consumed by the circuit the wattless current are, respectively:

(1) $\frac{1000}{\sqrt{2}}, 10$ (2) $\frac{50}{\sqrt{2}}, 0$ (3) 50, 0 (4) 50, 10

Solution: (1)

$$\text{Power} = \frac{v_0 I_0}{2}\cos\phi$$

$$= \frac{100 \times 20}{2} \cdot \cos\left(\frac{\pi}{4}\right) = \frac{1000}{\sqrt{2}} \text{ watt.}$$

$$I_{rms} = \frac{20}{\sqrt{2}} = 10\sqrt{2}\text{A}$$

$$I_{wattless} = I_{rms}\sin\phi = 10\text{A}.$$

86. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in the internal energy.

(1) (a) 195K (b) –2.7 kJ (2) (a) 189K (b) –2.7 kJ

(3) (a) 195K (b) 2.7 kJ (4) (a) 189K (b) 2.7 kJ

Solution: (2)

$$TV^{r-1} = \text{const.}$$

$$(300)(V)^{2/3} = T'(2V)^{2/3}$$

$$T' = \frac{300}{(2)^{2/3}} = 189\text{ K}$$

$$\Delta U = nC_v\Delta T = (2)\frac{3R}{2}(189 - 300)$$

$$= -2.7 \text{ kJ.}$$

87. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is:

(1) $\frac{Ka}{3mg}$ (2) $\frac{mg}{3Ka}$ (3) $\frac{mg}{Ka}$ (4) $\frac{Ka}{mg}$

Solution: (2)

$$B = -\frac{dP}{\left(\frac{dV}{V}\right)}$$

$$\frac{\frac{mg}{a}}{\left(\frac{dV}{V}\right)} = K$$

$\Rightarrow$ $\frac{dV}{V} = \frac{mg}{ka}$

and $\frac{dV}{V} = \frac{3dr}{r} = \frac{mg}{ka}$

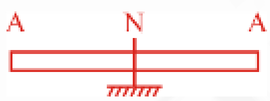
$\frac{dr}{r} = \frac{mg}{3ka}$.

88. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 2.5 kHz (2) 10 kHz (3) 7.5 kHz (4) 5 kHz

Solution: (4)

$$V = \sqrt{\frac{Y}{g}} = 5.85 \times 10^3 \text{ m/sec}.$$



$\frac{\lambda}{4} = 30$

$\Rightarrow$ $\lambda = 1.2$ m

$\nu = \frac{v}{\lambda} = 4.88 \times 10^3 \text{ Hz} \simeq 5 \text{kHz}$.

89. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly:

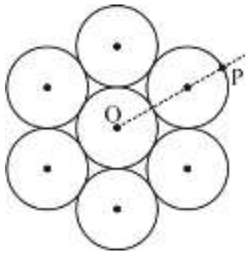
(1) 4.70×10^3 N/m^2 (2) 2.35×10^2 N/m^2
(3) 4.70×10^2 N/m^2 (4) 2.35×10^3 N/m^2

Solution: (4)

Force exerted $= (2mv \cos 45°)\, n$
$= 4.695 \times 10^{-1}$

$P = \frac{F}{a} = 2.34 \times 10^3$ N/m^2.

90. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertial of the arrangement about the axis normal to the plane and passing through the point P is:



(1) $\frac{55}{2}MR^2$

(2) $\frac{73}{2}MR^2$

(3) $\frac{181}{2}MR^2$

(4) $\frac{19}{2}MR^2$

Solution: (3)

$$I_0 = 6\left[\frac{MR^2}{2} + M(2R)^2\right] + \frac{MR^2}{2}$$

$$I_0 = \frac{55}{2}MR^2$$

$$I_p = I_0 + 7M\,(3R)^2$$

$$= \left(\frac{55}{2} + 63\right)MR^2$$

$$= \frac{181}{2}MR^2.$$